Watermarking of Compressive Sensed Measurements

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***Abstract*—In this paper, we propose a watermarking algorithm in order to embed a watermark w onto Compressive Sensed Measurements of a sparse signal x. Performance results are given for i) error-free channel, and ii) when the watermarked signal is subject to an additive white Gaussian noise. Our proposed decoding method outperforms both classical ℓ_2 and ℓ_1 minimization algorithms.**

I. Introduction

Compressive Sensing (CS) claims that a sparse signal can be reconstructed with far fewer random measurements with polynomial time algorithms using ℓ_1 minimization [1]. Channel decoding counterpart of the CS has been developed in [2] for linear decoding of a message from erroneous version under unbounded sparse noise. Sheikh and Baraniuk use this idea to embed watermark onto a sparse signal (e.g. DCT coefficients of an image) [3]. In this paper, we propose a watermarking scheme that embeds watermark directly onto CS measurements. This enables to embed watermark information while sensing.

II. Problem Statement

Let $x \in \mathbb{R}^N$ be a k-sparse signal and $w \in \{-1, 1\}^M$ be an M bit dense watermark message. The watermarked measurements $y \in \mathbb{R}^m$ are $y = Ax + Bw$, where A is $m \times N$ measurement matrix, B is $m \times M$ coding matrix generated by a secret seed, known to both encoder and decoder where $M < m \ll N$, and both A and B are full rank. The watermarked measurements can be altered by a malicious user or by channel imperfections

$$y_n = Ax + Bw + z, \tag{1}$$

where $z \sim \mathcal{N}(0, \sigma^2 I)$. Eq. (1) can be cast as $y_n = C \begin{bmatrix} x \\ w \end{bmatrix} + z$, where $C = [A|B]$. One classical solution can be ℓ_2 minimization such that

$$\begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} = C^{\mathrm{T}}(CC^{\mathrm{T}})^{-1} y_n, \tag{2}$$

where $^{\mathrm{T}}$ denotes the transpose operator. The other possible solution is to use $\ell_1$minimization of a $(k+M)$-sparse vector $[x \;\; w]^{\mathrm{T}}$ of length $N+M$, such that

$$\begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} = \arg\min_{[x \;\; w]^{\mathrm{T}}} \left\| \begin{bmatrix} x \\ w \end{bmatrix} \right\|_{\ell_1} \quad \text{s.t.} \quad \left\| y_n - C \begin{bmatrix} x \\ w \end{bmatrix} \right\|_{\ell_2} \le \epsilon_1. \tag{3}$$

Proposed decoding algorithm can be decomposed into two steps: a) We construct a matrix which annihilates the matrix B on the left, i.e., such that $FB = 0$. Then apply F to $y_n = Ax + Bw + z$, gives $\tilde{y} = F(Ax+Bw+z) = FAx + \tilde{z}$, where $\tilde{z} = Fz$. Note that FA is a fat matrix and it should satisfy the Restricted Isometry Property (RIP) in order to solve $\hat{x}$ with ℓ_1 minimization. In this paper, we choose A and B as random Gaussian matrices, hence FA will also satisfy the RIP with high probability. The estimate of the sparse signal is found via

$$\hat{x} = \arg\min_x \|x\|_{\ell_1} \quad \text{s.t.} \quad \|\tilde{y} - FAx\|_{\ell_2} \le \epsilon_2. \tag{4}$$

b) We estimate the watermark using ℓ_2 minimization as follows:

$$\hat{w} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}(y_n - A\hat{x}). \tag{5}$$

III. Experimental Results

We embed M bit length watermark on to $k = 30$-sparse signal of length $N = 512$ using $m = 145$ measurements. We look for maximum achievable embedding rate $\mathcal{R} = M/m$ in bits/measurement when $P(w \neq \mathrm{sgn}(\hat{w})) \to 0$. In the meantime, we are interested in a low mean reconstruction ℓ_2 error $\mathbb{E}\{\|x - \hat{x}\|_2/\|x\|_2\}$. Being x, w, A, B randomly generated in each iteration, we ran 250 iteration for different M values. Sparse signal x is generated as $x_{\Lambda_i} \sim \mathcal{N}(0, 25)$, where x_{Λ_i} is i th non-zero coefficient of x. For $\sigma = 0$, Fig. 1 claims $\mathcal{R} \le 45/145$ for the proposed method which outperforms the classical methods discussed above. Fig. 2 shows for maximum achievable rate of proposed method, we have also better performance on reconstruction $\mathbb{E}\{\|x - \hat{x}\|_2/\|x\|_2\}$. Under Gaussian noise with $\sigma = 0.15$, Fig. 3 claims $\mathcal{R} \le 40/145$ for the proposed method which also outperforms the classical methods. Furthermore, Fig. 4 claims, under Gaussian noise, mean reconstruction ℓ_2 error $\mathbb{E}\{\|x - \hat{x}\|_2/\|x\|_2\}$ of proposed algorithm is less than or equal to ℓ_1 decoding performance.

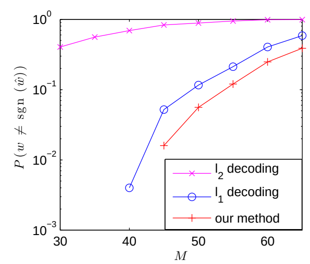

Fig. 1. M vs. $P(w \neq \mathrm{sgn}(\hat{w}))$ for $\sigma = 0$.

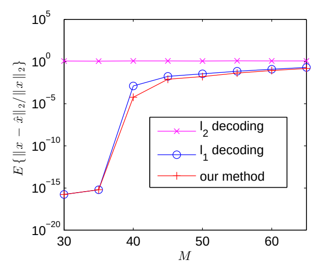

Fig. 2. M vs. $\mathbb{E}\{\|x - \hat{x}\|_2/\|x\|_2\}$ for $\sigma = 0$.

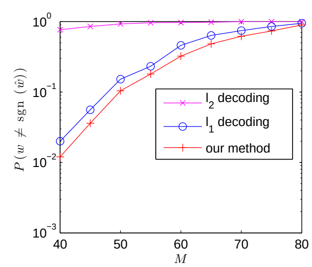

Fig. 3. M vs. $P(w \neq \mathrm{sgn}(\hat{w}))$ for $\sigma = 0.15$.

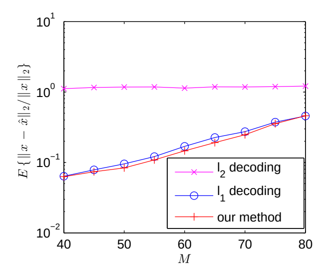

Fig. 4. M vs. $\mathbb{E}\{\|x - \hat{x}\|_2/\|x\|_2\}$ for $\sigma = 0.15$.

References

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